## Originally published as:

**Leimbach, M., Schultes, A., Baumstark, L., Giannousakis, A., Luderer, G. (2017):**
Solution algorithms for regional interactions in large-scale integrated assessment models of climate change. - Annals of Operations Research, 255, 1-2, 29-45

**DOI:** 10.1007/s10479-016-2340-z

**Noname manuscript No.**
(will be inserted by the editor)

# Solution algorithms for regional interactions in large-scale Integrated Assessment models of climate change

**Marian Leimbach* · Anselm Schultes* · Lavinia Baumstark · Anastasis Giannousakis · Gunnar Luderer**

the date of receipt and acceptance should be inserted later

**Abstract** We present two solution algorithms for a large-scale integrated assessment model of climate change mitigation: the well known Negishi algorithm and a newly developed Nash algorithm. The algorithms are used to calculate the Pareto-optimum and competitive equilibrium, respectively, for the global model that includes trade in a number of goods as an interaction between regions. We demonstrate that in the absence of externalities both algorithms deliver the same solution. The Nash algorithm is computationally much more effective, and scales more favorably with the number of regions. In the presence of externalities between regions the two solutions differ, which we demonstrate by the inclusion of global spillovers from learning-by-doing in the energy sector. The non-cooperative treatment of the spillover externality in the Nash algorithm leads to a delay in the expansion of renewable energy installations compared to the cooperative solution derived using the Negishi algorithm.

**Keywords** climate change mitigation · general equilibrium · non-cooperative solution · non-linear programming · trade interaction · energy system modeling

**Acknowledgements** Funding from the German Federal Ministry of Education and Research (BMBF) in the Call Economics of Climate Change (funding code 01LA1105A CliPoN) is gratefully acknowledged.

Marian Leimbach · Anselm Schultes · Lavinia Baumstark · Anastasis Giannousakis · Gunnar Luderer
Potsdam Institute for Climate Impact Research, P.O.Box 601203, 14412 Potsdam, Germany. E-mail: leimbach@pik-potsdam.de

Anselm Schultes
Technische Universitaet Berlin, Secretariat EB 4-1, Strasse des 17. Juni 145, 10623 Berlin, Germany.

* Both first authors contributed equally to this work.

## 1 Introduction

Climate change is a complex global phenomenon involving long time scales. The assessment of climate change mitigation policies requires specialized tools to deal with the long-term impacts and the interactions between different environmental and socio-economic systems. Over the last three decades, Integrated Assessment (IA) models have increasingly been used for analyzing climate change mitigation and adaptation strategies. IA models describe the complex relations between macro-economic, energy, climate, and land use systems. Yet, the solution algorithms used to solve these large-scale models are hardly ever discussed in the literature (Nordhaus and Yang 1996). The choice and specification of the solution algorithm determine, on the one hand, the numerical effectiveness of the solution process. On the other hand, the type of regional interactions covered in IA model places requirements on the solution algorithms.

In this paper we discuss different solution algorithms of an IA model that can be classified as a policy optimization model (Weyant et al. 1996). We present a new implementation of an algorithm that is commonly used to compute a competitive equilibrium solution in less complex models. This new algorithm is computationally more effective than previously used algorithms and allows for integration of real-world externalities that are usually not covered by existing IA models.

In the discussion of the solution algorithms, we are not interested in the details of the underlying mathematical optimization procedures. Instead, we describe the algorithms with emphasis on economic mechanisms, asking how to reconcile actions and decisions of different actors. Our model does not capture regional interaction due to climate change damages. Therefore, we do not contribute to the literature addressing climate policies and the stability of climate coalitions based on the climate externality (Finus et al. 2014). The new algorithm developed in this contribution primarily captures international trade. Recent literature increasingly focuses on the interaction of trade and climate policies (e.g. Copeland and Taylor 2005, Weber and Peters 2009). Trade is relevant to climate policy, as, for example, climate policies may put revenues of fossil fuel exporters at risk. Models including trade interactions are useful to assess viable mitigation strategies of regions or countries and provide insights into terms-of-trade effects.

The paper is structured in the following way: Referring to the existing literature, we discuss in Section 2 the representation of regional interactions and externalities in IA models, and relevant solution algorithms. In Section 3, we introduce the IA model REMIND. Two different algorithms to find a general equilibrium solution are presented in Section 4, highlighting advantages of the new algorithm. The application of the model for the evaluation of a climate change mitigation scenario in Section 5 highlights differences in the solution of the two algorithms due to the inclusion of an externality. This indicates the potential of the newly developed algorithm in analyzing other policy questions. We end with conclusions in Section 6.

## 2 Regional interactions in Integrated Assessment models

IA models simulate long-term dynamics of the socio-economic system, often including the energy sector and parts of the environmental system, in particular the climate system. IA models of the policy optimization type, for example MERGE (Manne et al. 1995), DICE (Nordhaus and Boyer 2000), and MESSAGE (Messner and Schrattenholzer 2000), derive a solution in form of mitigation strategies by cost minimization or welfare maximization. The solution algorithms for these models either treat the interaction of decentralized regional actors explicitly, or assume a global social planner – this differences is most important in the presence of externalities. While a global social planner internalizes external effects between regions, decentralized actors do usually not anticipate and internalize them. In IA models, these actors commonly are world regions. We distinguish four types of regional interactions:

(I) For trade interactions, the synchronization of trade decisions is the modeling challenge in a decentralized decision framework, as is the redistribution of wealth in a social planner framework. Trade relations are therefore commonly considered in a restricted way in IA models only, for example excluding capital trade and changes in the current accounts (Bosetti 2006).

(II) Technological learning in energy technologies is included in some IA models with great detail in the energy system. Learning effects decrease investment costs driven by cumulative investments in modern energy conversion technologies (i.e. learning-by-doing) (Manne and Richels 2004, Rao et al. 2006, Magne et al. 2010). Learning effects are partly external, as some investment costs decrease independently of where and by whom the new capacities are built.

(III) Technological spillovers affecting the productivity of production factors are taken into account just by few IA models (Crassous et al. 2006, Bosetti et al. 2008, Leimbach and Baumstark 2010, Huebler et al. 2012). They depend on R&D investments and may represent externalities in a way similar to learning spillovers.

(IV) The climate externality is caused by the global impact of GHG emissions of each country. If the climate externality is included in the form of a damage function or a global climate target, it is fully internalized by a social planner. In a decentralized setting, however, depending on whether or not the actors internalize this externality, a cooperative solution or a non-cooperative solution can be obtained (Brechet et al. 2014). Nordhaus and Yang (1996) prominently discuss the difference of the cooperative and non-cooperative solution, including algorithmic implications. Their results show that non-cooperative policy, with selfishly acting nations ignoring spillovers and damages imposed on others, leads to much smaller emission reductions than cooperative policy.

The representation of these regional interactions in IA models varies, and so does the specification of the solution algorithm. A complete overview is beyond the scope of the paper – we focus on a particular IA model, the REMIND model. Regarding the four regional interactions discussed above, REMIND includes trade on different markets. Interactions through global technological learning are covered by the model as well. By contrast, R&D spillover externalities and the climate externality are not represented in REMIND. While the Nash and Negishi algorithm, as presented in the next section, compute the same solution with respect to the trade interaction, they compute a non-cooperative (Nash) and cooperative (Negishi) solution with respect to the learning externality.

REMIND belongs to the class of intertemporal optimization models which are computationally expensive (Pan 1992), and is formulated as a non-linear optimization problem. The computation time of such problems rises strongly with the number of variables in the problem. The main challenge in solving the REMIND model is the large number of different markets that have to be cleared simultaneously. We apply two different algorithms to compute a competitive equlibrium. One follows a proven approach for finding a Pareto-optimal solution that corresponds to a competitive equilibrium: Negishi (1972) demonstrated that this market equilibrium can be computed by global welfare optimization with a particular adjustment of the regional welfare weights (see Section 4). The other algorithm in this paper – the Nash algorithm – follows a Walrasian type price adjustment approach. The basic idea of this type of algorithm dates back to the tatonement process of the Walrasian auctionneer, as described by Arrow and Hahn (1971). By iterative price adjustments, this algorithm searches for a fixed point where the aggregated excess demand equals zero. Numerical determination of the equilibrium of a Walrasian system was first provided by Scarf (1967). Other essential contributions to the computation of equilibria in large-scale models are, for example, Dixon (1975) and Manne and Rutherford (1994).

The Negishi algorithm used in REMIND to find a Pareto-optimal solution is similar to that in RICE (Nordhaus and Yang 1996) and MERGE (Manne et al. 1995). The Nash algorithm we implement does not have direct antecedents in IA models: While Nordhaus and Young (1996) implemented a Nash algorithm[1] in order to find a non-cooperative solution with respect to the climate externality, we develop an algorithm for a competitive equilibrium solution with respect to the trade interaction.

## 3 REMIND Model

REMIND is a global energy-economy-climate model spanning the time from 2005 to 2100 (Leimbach et al. 2010). Its general structure is illustrated in Fig. 1. A comprehensive description of REMIND can be found in Luderer et al. (2015).

The macro-economic core of REMIND is a Ramsey-type optimal growth model where intertemporal welfare is maximized. The world is divided into eleven model regions. Each region is modeled as a representative household with a utility function $U(r)$ that depends only on per-capita consumption:

[1] Within an iterative sequence Nordhaus and Yang (1996) compute a closed-loop Nash equilibrium (Fudenberg and Levine 1988). Technically, this is an outcome of a finite game with perfect information and calculated through backward induction (Mas-Colell 1995, Chapter 9, Kicsiny et al. 2014). A survey on Generalized Nash equilibrium problems is provided by Facchinei and Kanzow (2010). Yang (2003) demonstrated how the closed-loop solution can be found by a computationally more tractable sequence of open-loop equilibria.

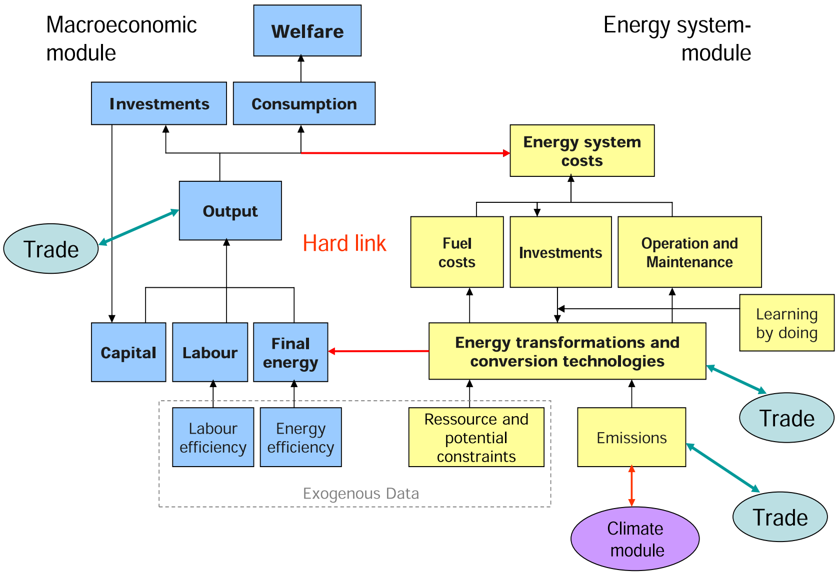


**Fig. 1** Structure of REMIND

$$U(r) = \sum_{t=t_0}^{T} (1+\zeta)^{-t} \cdot L(t,r) \cdot \ln\left(\frac{C(t,r)}{L(t,r)}\right) \qquad \forall\ r. \tag{1}$$

The variable $C(t,r)$ is the consumption in time-step $t$ and region $r$, $L(t,r)$ is population, and $\zeta$ is the rate of pure time preference.

Each region generates macro-economic output (GDP) based on a calibrated and nested constant elasticity of substitution (CES) production function with factor inputs labor, capital, and final energy. Final energy is generated through a detailed representation of the energy system in a nested CES tree. The production function reads:

$$V_{out}(t,r) = \left(\sum_{M_{CES}} (\theta_{in} V_{in}(t,r))^{\rho_{out}}\right)^{\dfrac{1}{\rho_{out}}} \qquad \forall t, r, out. \tag{2}$$

$V_{out}(t,r)$ represents CES function output (GDP and intermediate products) and $V_{in}(t,r)$ CES function input. The mapping $M_{CES}$ assigns input types $in$ to each output $out$. $\theta$ and $\rho$ are parameters representing efficiency and elasticity of substitution, respectively. GDP $Y(t,r)$ is available for consumption, investments into the macro-economic capital stock $I(t,r)$, energy system expenditures $E(t,r)$ and for the export of composite goods $X_G(t,r)$, which may instead also be imported as $M_G(t,r)$:

$$Y(t,r) = C(t,r) + I(t,r) + X_G(t,r) - M_G(t,r) + E(t,r) + D(t,r). \tag{3}$$

The term $D(t, r)$, as defined in Eq.(15), is relevant only for technical reasons of the convergence process and does not change the solution point of the model.

Macro-economic investments $I(t, r)$ as control variable enter a common capital stock equation for macro-economic capital $K(t, r)$ with depreciation rate $\delta$:

$$K(t+1, r) = (1-\delta) \cdot K(t, r) + I(t, r) \qquad \forall r, t. \tag{4}$$

Energy system expenditures $E(t, r)$ consist of investment costs $E_{inv}(t, r)$, fuel costs $E_{fc}(t, r)$ as well as operation and maintenance costs $E_{om}(t, r)$:

$$E(t, r) = E_{fc}(t, r) + E_{om}(t, r) + E_{inv}(t, r) \qquad \forall r, t. \tag{5}$$

By means of energy expenditures and final energy input in the production function, the macro-economic core and the energy system module are hard-linked to each other. This ensures simultaneous equilibria on all primary energy carrier and capital markets (Bauer et al. 2008). Around fifty different technologies are represented in REMIND for the conversion of primary energy into secondary energy carriers as well as for the distribution of secondary energy carriers into final energy. Technology choice follows implicit cost optimization based on investment costs, operation and maintenance costs, fuel costs, emission costs, efficiencies, lifetimes, and learning rates. Endogenous technological progress is captured by learning-by-doing: Variable investment costs of low-carbon technologies decrease with cumulative installed capacity through global learning curves.

Balance equations ensure that energy demand matches production. One balance equation equals production $P_e(t, r)$ of primary energy type $e$ to extraction $F_e(t, r)$ and net export $X_e - M_e$ of the corresponding primary energy carrier:

$$P_e(t, r) \quad = \quad F_e(t, r) \quad - \quad X_e(t, r) \quad + \quad M_e(t, r) \qquad\qquad \forall \quad t, r, e \tag{6}$$

Resource scarcity is reflected by extraction cost curves. The model accounts for $CO_2$ emissions from fossil fuel combustion and land use as well as emissions of other greenhouse gases (GHGs). REMIND is coupled to the MAGICC climate model (Meinshausen et al., 2011) to translate emissions into changes in atmospheric composition, radiative forcing, and temperature increase.

Besides macroeconomic investment, investments into the installation of energy conversion capacities, resource extraction, and the reduction of non-energy related greenhouse gases, trade is another control variable of the regional actors. Trade between regions is induced by differences in factor endowments and technologies. There is trade in five primary energy carriers (oil, coal, gas, biomass, uranium), in a composite good (aggregated output), and in emission permits. Trade is not bilateral, but through exports into and imports from a common pool. Capital mobility is represented by free trade in the composite good and by the possibility of intertemporal trade – resulting in comparatively high capital trade flows. Nordhaus and Yang (1996) and Leimbach et al. (2015) discuss alternatives approaches to intertemporal trade. Capital mobility and intertemporal trade cause price equalization and guarantee an intertemporal and inter-regional equilibrium.

## 4 Solution algorithms

To find the intertemporal and inter-regional equilibrium, a mechanism of reconciling trade decisions of regions is needed. In the following subsections we describe the Negishi and the Nash algorithms in detail.

### 4.1 Negishi algorithm

We apply a sequential joint maximization algorithm (Manne and Rutherford, 1994) – also called the Negishi algorithm (Negishi, 1972). In this iterative algorithm, the objective functions of the individual regions $U(r)$, as defined in Eq. (1), are aggregated into a global objective function $W$ by means of welfare weights $w^i(r)$ (index i is the iteration index):

$$W = \sum_r w^i(r) \cdot U(r). \tag{7}$$

Market clearing is included as a constraint to the optimization problem. With $X_j(t,r)$ and $M_j(t,r)$ as export and import of region $r$ in period $t$, the following clearance condition holds for each market $j$:

$$\sum_r (X_j(t,r) - M_j(t,r)) = 0 \quad \forall\ t, j. \tag{8}$$

A distinguished Pareto-optimal solution, which in the absence of externalities also corresponds to a competitive market solution, is obtained by iteratively adjusting the welfare weights based on the intertemporal trade balances $B^i(r)$:

$$B^i(r) = \sum_t \sum_j p_j^i(t) \cdot [X_j^i(t,r) - M_j^i(t,r)] \quad \forall\ r, i. \tag{9}$$

Present value world market prices $p_j^i(t)$ are derived iteratively as shadow prices from Eq. 8.

A new set of weights is derived iteratively (Leimbach et al. 2015):

$$w^{i+1}(r) = w^i(r) + \frac{B^i(r)}{\sum_t (1+\zeta)^{-(t-t_0)} L(t,r)} \quad \forall\ r, i. \tag{10}$$

Based on the new weights, a new solution is computed from which we derive $B^{i+1}(r)$. This algorithm has a fixed point in which the intertemporal trade balance of each region converges towards zero. We stop the iteration as soon as the residual deviation from zero is sufficiently small. An exemplary convergence process is shown in Fig. 2.

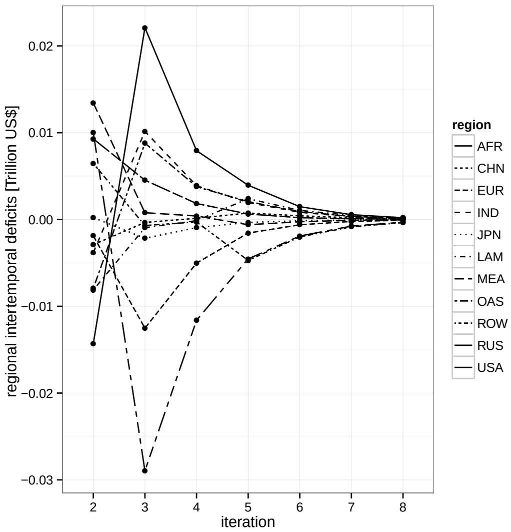


**Fig. 2** Exemplary convergence process generated by the Negishi algorithm. Deficits from the intertemporal trade balance (9) for each region approach zero over some 8 iterations.

## 4.2 Nash algorithm

The Nash algorithm assumes that decisions are taken by decentralized regional actors. In contrast to the Negishi algorithm it neither includes a global welfare function nor explicit market clearing conditions as part of the optimization. Instead, the optimization of each region directly includes its respective intertemporal budget constraint:

$$B(r) = \sum_{t,j} p_j^i(t) \cdot (1 + \chi^i) \cdot [X_j^i(t,r) - M_j^i(t,r)]. \tag{11}$$

The factor $\chi^i$ is only of importance for technical aspects of the algorithm, and is explained in Eq. (14).

Market clearing is achieved through a Walrasian-auctioneer type iterative price adjustment. Regional actors start from an initial price vector and choose their trade pattern, acting as price takers. The regional solutions are consequently collected, and the price for the next iteration is adjusted based on the surplus $S_j^i(t)$ on each market:

$$S_j^i(t) = \sum_r \left( X_j^i(t,r) - M_j^i(t,r) \right) \tag{12}$$

$$p_j^{i+1}(t) = p_j^i(t) \left( 1 - \eta_j H_j^i(t) \frac{S_j^i(t)}{Z_j^i(t)} \right). \tag{13}$$

Market surpluses are normalized by $Z_j^i(t)$, the global consumption of the respective good, which is a proxy measure for the potential market volume. The parameters $\eta_j^{-1}$ play the role of price elasticities, and are set from experience as to support convergence and minimize convergence time. An auxiliary parameter $H_j^i(t)$ is used to introduce a time-dependence into the price elasticity $\eta_j^{-1}$ that is set from experience in order to achieve convergence.

The algorithm converges towards a fixed point where markets clear, as demonstrated exemplarily in Fig. 3. We stop the iteration as soon as the market surplus falls below a certain residual threshold, which typically requires in between 30 and 100 iterations.

The biggest challenge with this formulation of the algorithm is the large number of markets that need to be cleared simultaneously. In our model there is one market for each traded good at each time step. In order to guarantee convergence, we employ two auxiliary mechanisms: Both act as guardrails protecting against a too abrupt change in trade patterns across iterations that may otherwise lead to diverging trade patterns. These mechanisms do not influence the solution point of the iteration. The first one allows regions to anticipate price changes caused by their trade decisions endogenously within the optimization. This is achieved by an additional factor $\chi^i$ in the intertemporal budget equation (11):

$$\chi^i(t,r) = \xi_j^i \frac{\left([X_j^{i-1} - M_j^{i-1}] - [X_j^i - M_j^i]\right)}{V_j^i(t,r)} \tag{14}$$

This factor is linear in the differences between the net exports of subsequent iterations (we suppressed the time and region index of the trade variables). An interpretation of the anticipation mechanism from an economic perspective is: Regions are able to anticipate a decline (increase) in the price for a good when increasing their exports (imports), enabling strategic behavior on the markets (Mandel and Gintis 2016). Technically, the mechanism helps the algorithm to converge.

In order to prevent the fixed point of the algorithm to be influenced by this kind of strategic behaviour, we smoothly fade out the anticipation parameter $\xi^i$ to zero as soon as the markets are reasonably close to clearance. We make sure that the influence of the price anticipation helper mechanism on the solution point is negligible: Varying the anticipation parameter in numerical experiments, we observe a robust solution point. The variation in the solution due to the anticipation parameter is much less than the already small variance that is due to residual market surpluses, which not only depend on the length of the convergence process but also on the starting point. As there are non-convexities in the model, multiple equilibria may in principle exist. In the course of our experiments we only observe unique solutions though, disregarding the aforementioned inherent small numerical variance.

The second auxiliary mechanism is a penalty cost $D(t,r)$ depending on the change in the regional trade pattern over iterations, a mechanism sometimes referred to as regularization. The square of the deviation from last iteration's trade pattern is multiplied by a weight parameter $\Omega_j$, and priced into each respective region's budget equation (3) as a penalty:

$$D(t,r) = \sum_j \Omega_j \frac{p_j^i(t)}{V_j^i(t,r)} \left( [X_j^{i-1} - M_j^{i-1}] - [X_j^i - M_j^i] \right)^2 \qquad \forall\ t,r \tag{15}$$

Regions are thus prevented from changing their trade pattern all too abruptly across iterations. As trade patterns converge, this cost penalty goes to zero and does thus not influence the solution point of the algorithm.

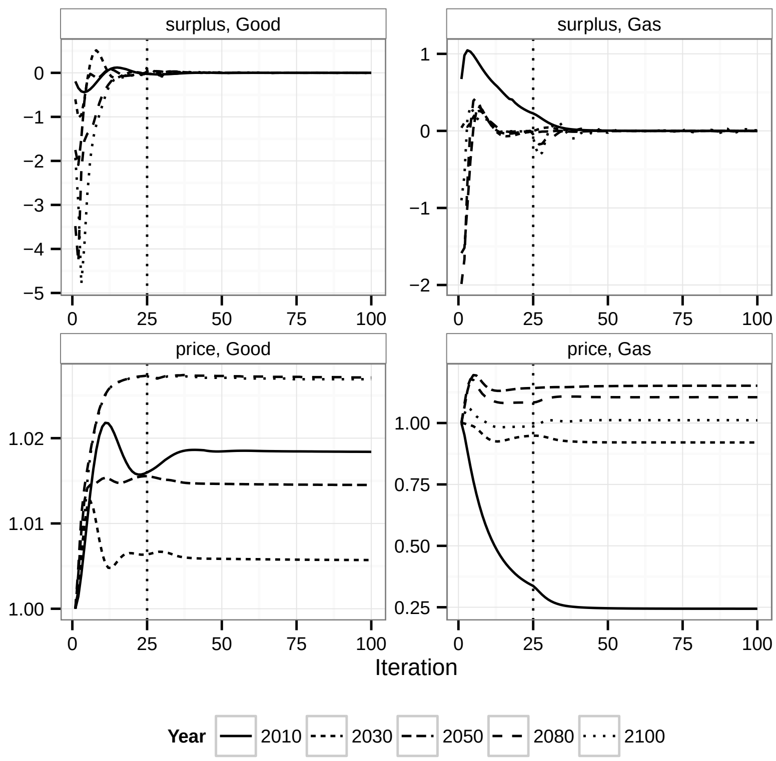


**Fig. 3** Exemplary convergence process generated by the Nash algorithm. Residual surplus over iteration are in the first row of panels, and prices over iterations in the second row are shown for two markets (Goods and Gas) at selected time steps. Units for surplus are Trillion US Dollar for goods, and exajoule per year for the gas market. Prices are normalized to their first-iteration value. The begin of the phase-out of the price anticipation mechanism is marked by a dotted vertical line.

### 4.3 Computational effectiveness

Both solution algorithms, Negishi and Nash, are implemented in GAMS (Brooke et al. 1992). We use CONOPT3 (Drud 1994) as the solver for the non-linear programming problem (NLP) of the global social planner maximizing global welfare (Eq. 7) in the Negishi case, and the regional social planners maximizing regional welfare (Eq. 1) in the Nash case, respectively. The two respective NLP problems differ in their number of non-superbasic variables by a factor of $m = 11$, the number of regions in our model. From our experience, the computing time to solve NLP models grows much faster than linear with the number of non-superbasic variables. This can be understood assuming matrix inversion is the most expensive operation within the solver, which scales polynomially with the size of the system (Pan 1992).

Large-scale NLP models (size of more than 100 000 variables and equations) in GAMS often cannot be reliably solved without providing a starting point for the solver in form of a GDX file. The choice of the GDX file from previous model runs significantly influences model run time. In order to compare performance between the Nash and Negishi algorithms, we set up the following model runs: Two exemplary REMIND scenarios, a business-as-usual and a carbon tax scenario are each run in Negishi-, sequential-Nash-, and parallel-Nash-mode. We use six different GDX points as initial starting points for the solver, some known to be closer to the solution, some farther away. Run times of the resulting 36 model realizations are shown in Fig. 4. The median run times are significantly smaller for the Nash algorithm, around an order of magnitude below the Negishi value. Furthermore, the performance of the Nash algorithm is less dependent on the starting point, as seen from the smaller spread of run times in the Nash mode.

Computational effectiveness mainly depends on three factors: First, for NLP problems the time complexity of problems typically rises much faster than linear with the size of the problem. Formulating the model as $m$ independent regional NLP problems thus potentially decreases the total run time, even if the problems are solved sequentially.

Second, the Nash algorithm requires an adjustment of many more parameters in between iterations than the Negishi algorithm – potentially increasing total run time. In the Nash algorithm, prices on markets for all goods and all times (on the order of 100) have to be adjusted in each iteration, while the Negishi algorithm only requires updating the Negishi weight of each region (11 in our case). Consequently, while the Negishi algorithm takes about 10 iterations to converge, the Nash algorithm requires in between 30 and 100 iterations. In our implementation, the first effect outweighs the second one quite drastically. As seen from Fig. 4, the median run time in sequential Nash mode (where all regions are solved on the same CPU core sequentially) is 4.2 hours, much less than the 42.4 hours in Negishi mode.

Third, the separate regional problems can be solved in parallel in the Nash mode, using the GAMS grid computing facilities. Each regional problem then runs as a separate thread, allowing for the distribution of these threads on different CPU cores[2]. This contributes to the reduction in total run time of the model in Nash

[2] We solve the model on a high-performance computer cluster equipped with Intel Xeon E5472 CPUs at 3.0GHz.

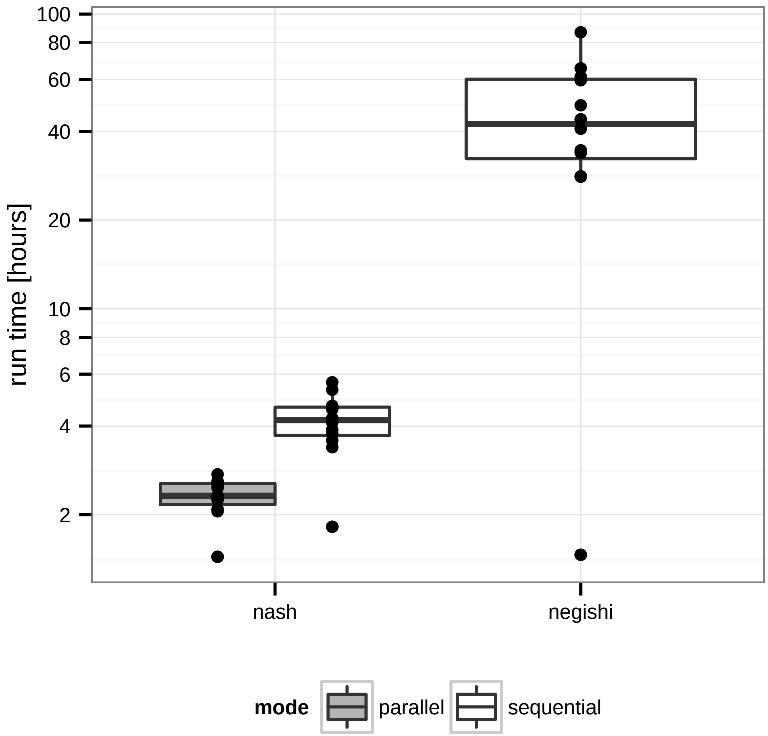


**Fig. 4** Comparison of run times (wall clock time) of Nash and Negishi algorithms over a set of different starting points (GDX) and scenarios. In the Nash case, sequential- and parallel-mode results are shown. Boxes indicate interquartile range, the horizontal lines the median value.

mode, reducing the median run time from 4.2 hours in sequential Nash mode to 2.3 hours in the parallel mode.

The separation into regional NLP problems in the Nash algorithm also allows for a very favorable scaling of run time with the number of regions in the model. Given the GAMS grid computing facility has access to as many cores as regions in the problem, total run time should not significantly increase with the number of regions. This is in stark contrast to the scaling behaviour of the Negishi algorithm, where the number of regions is effectively limited by the drastic increase of run time of the single NLP problem with its size.

### 4.4 Equivalence in the absence of externalities

In the absence of the learning-by-doing externality (i.e. assuming that there are no learning technologies), trade is the only interaction between regions in our model. In this case, the Negishi and Nash solution algorithms converge to the same fixed point – the competitive equilibrium. We demonstrate this by comparing both solutions numerically in detail.

| Trade pattern deviation [%] | median | mean | max |
|---|---|---|---|
| Oil | 0.05 | 0.17 | 2.01 |
| Coal | 0.08 | 0.34 | 5.31 |
| Gas | 0.12 | 0.40 | 5.80 |
| Biomass | 0.56 | 0.99 | 2.82 |
| Uranium | 0.09 | 0.91 | 12.84 |
| Composite good | 0.05 | 0.11 | 1.45 |

**Table 1** Deviation of trade patterns between Nash and Negishi solution on different markets in relative terms.

| Trade pattern deviation relative to global consumption [%] | median | mean | max |
|---|---|---|---|
| Oil | 0.000 | 0.004 | 0.069 |
| Coal | 0.003 | 0.020 | 0.181 |
| Gas | 0.003 | 0.039 | 0.754 |
| Biomass | 0.001 | 0.013 | 0.875 |
| Uranium | 0.007 | 0.333 | 9.033 |
| Composite good | 0.000 | 0.000 | 0.009 |

**Table 2** Deviation of trade patterns between Negishi and Nash solution on different markets, relative to the global consumption of the respective market good in percent.

To compare the two solutions, statistics on relative deviations in regional trade patterns between the two solution points are shown in Tab. 1. Statistics are based on the relative deviations of net exports at all time periods and regions of an exemplary Nash and Negishi run:

$$\text{Trade pattern deviation} = 100 \cdot \left| \frac{X(t,r)^{\text{nash}} - M(t,r)^{\text{nash}}}{X(t,r)^{\text{negishi}} - M(t,r)^{\text{negishi}}} - 1 \right| \tag{16}$$

We exclude the resource markets with a very small volume of below 0.5 EJ/yr from this analysis, as the resulting high relative deviations would only be an artifact of regions switching from import to export of this specific resource. The deviations are small in general, with median relative deviations of below 0.6% across all markets and all times. The relatively high maximum deviations on the uranium markets of up to 13% are due to the high flexibility (indeterminacy) of uranium deployment across regions in the model as compared to other types of primary energy supply.

We introduce a more aggregated measure for the residual deviations in the trade structure of the two solutions: The deviation of trade patterns between Nash and Negishi solution, divided by to the global consumption of the respective good or primary energy type, as shown in Tab. 2. These normalized deviations in trade patterns are very small, with a median value below 0.007%.

Differences in the regional consumption paths are also small, with a maximum deviation of around 0.07% and a median of 0.02%.

## 5 Application to climate change mitigation

In this section, we present an exemplary climate change mitigation assessment with REMIND using both solution algorithms discussed previously. REMIND does not model climate change damages, but evaluates mitigation strategies for a given global climate target, that is, the analysis here is a cost-effectiveness, not a cost-benefit analysis. The cost-efficient allocation of mitigation efforts is based on a globally uniform carbon tax imposed on each region. The tax path is iteratively adjusted based on the difference between the simulated radiative forcing and and the forcing level corresponding to the aspired climate stabilization target (see below), ensuring that the given climate target is achieved.

The alternative use of the Nash and Negishi algorithm computes a constrained competitive equilibrium solution and a constrained Pareto-optimum, repectively. The solutions deviate from each other due to a different handling of technological spillovers. In contrast to the experiment in the previous section, we activated the technological learning externality for this application. Due to learning-by-doing, the globally uniform specific investment costs $I_L(t)$ decrease with the cumulative capacities $\mathrm{CC}_L(t,r)$ for emerging low carbon energy conversion technologies of type $L$:

$$I_L(t) = I_{L,\mathrm{floor}} + I_{L,0}\left(\sum_r \mathrm{CC}_L(t,r)\right)^{-b_L} \tag{17}$$

$I_{L,\mathrm{floor}}$ and $I_{L,0}$ represent the floor costs and the initial variable costs of investments, respectively. The parameter $b_L$ describes the learning rate of technology $L$. Learning technologies are solar photovoltaics, concentrated solar power, wind power, hydrogen cars, electric cars, and storage technologies.

The global social planner of the Negishi algorithm anticipates that regional investments into learning technologies reduce the respective investment costs worldwide. The respective regional social planners in the Nash algorithm do not take these spillovers to other regions into account in their investment decision, creating a wedge between the Negishi solution and the Nash solution. Regarding the learning externality, a cooperative solution is computed by the Negishi algorithm and a non-cooperative solution by the Nash algorithm. Global technological learning still exists in the decentralized world though. In each iteration of the Nash algorithm, the unanticipated spillover effect is captured through the inclusion of the investment decisions of the decentralized actors from the preceding iteration.

The international community has agreed on the long-term target of limiting global warming to no more than 2°C above pre-industrial levels. Here we consider a climate mitigation scenario with a radiative forcing target of 2.6 W/m$^2$ by 2100, allowing for temporary overshoot of this target during the century. Such scenarios have been shown to keep global warming below 2°C with a high likelihood (Clarke et al. 2014, Rogelj et al. 2011). This climate target requires a drastic reduction of global greenhouse gas emissions and a sustained transformation of the global energy system. Fig. 5 shows the emission trajectories for both, the Negishi and the Nash solution, for a baseline scenario with no mitigation (scenario names: NE-

BAU, NA-BAU), and a mitigation scenario (NE-450 and NA-450) simulated by REMIND.

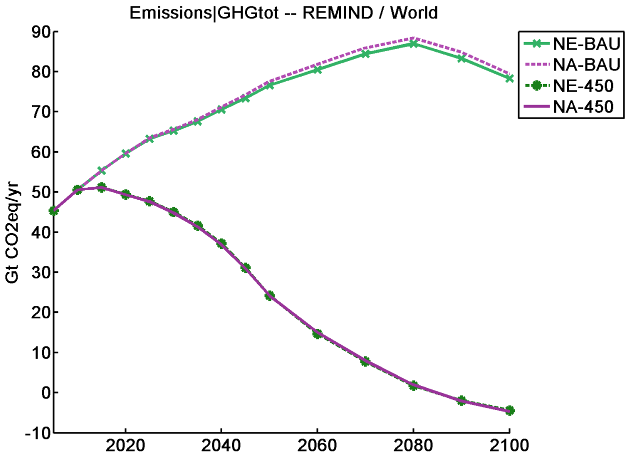


**Fig. 5** Total greenhouse gas emissions over time for different scenarios: Baseline scenarios (named "BAU"), and climate policy scenarios in line with the 2°C target ("450"), each for the Nash ("NA-") and Negishi("NE-") algorithm.

While the baseline scenarios show an increase of total GHG emissions until 2080 to a level of around 90 $GtCO_2eq$, emissions peak at 51 $GtCO_2eq$ in 2015 in the mitigation scenarios and decline to below zero over the second half of the century. The emission trajectories differ only slightly between the Nash and Negishi solution. While learning technologies play a minor role in the baseline scenario, they are heavily used in the mitigation scenario, but the optimal emission trajectory is mainly determined by the climate target.

The consumption of primary energy and electricity in 2050, as shown in Fig. 6 and Fig. 7, indicates the marked transformation of the energy system induced by the ambitious mitigation policies. In the baseline scenarios, fossil fuels are still dominating the energy mix in 2050. In the mitigation scenarios, by contrast, the high share of renewables increases constantly until 2100 – modern biomass in the non-electricity sector and solar and wind in the electricity sector. Furthermore, energy efficiency improvements contribute significantly to emissions reduction. Primary energy consumption is reduced by around 20% in 2050 and more than 30% in 2100 compared to the baseline scenario. This reduction is mainly at the expense of the non-electricity sector, while the share of electricity on final energy consumption increases continuously.

Differences between the Nash and Negishi solution are moderate. Within the baseline scenario, the reduced incentive to invest into learning technologies results in hardly any investments in solar technologies until 2050 in the Nash solution. Some investments in solar technologies is found for the Negishi solution (Fig. 8). Differences can be seen in the use of solar energy in the mitigation scenario. A substantial

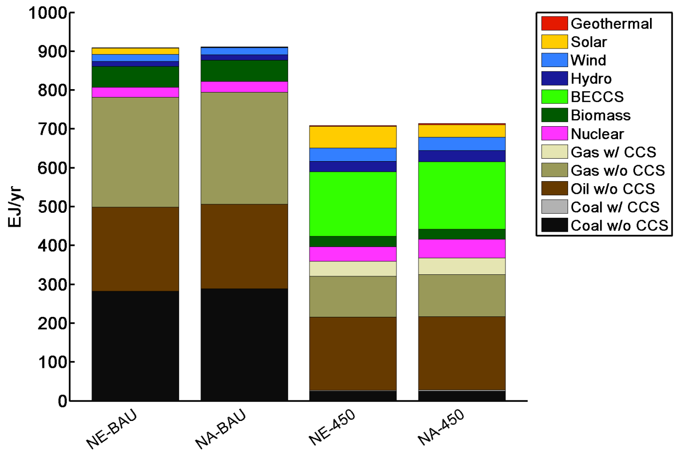


**Fig. 6** Primary energy consumption in 2050 for different scenario: Baseline scenarios (named "BAU"), and climate policy scenarios in line with the 2°C target ("450"), each for the Nash ("NA-") and Negishi("NE-") algorithm.

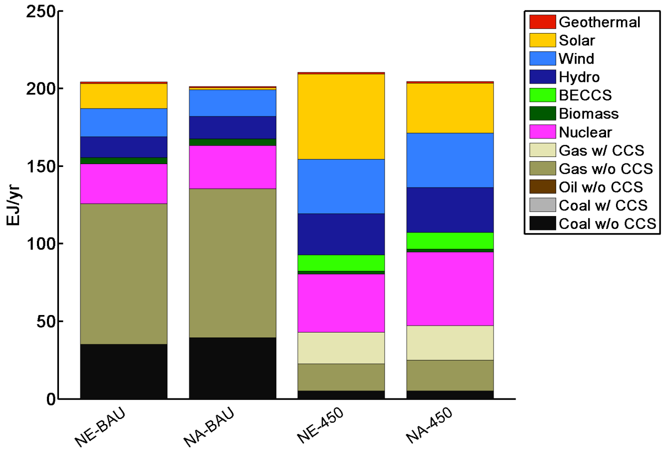


**Fig. 7** Electricity consumption in 2050

part of nuclear energy that is used for electricity production in the Nash solution is replaced by solar energy in the Negishi solution (Fig. 7). As a common pattern, the expansion of solar technologies is delayed by around 10-15 years in the non-cooperative Nash solution.

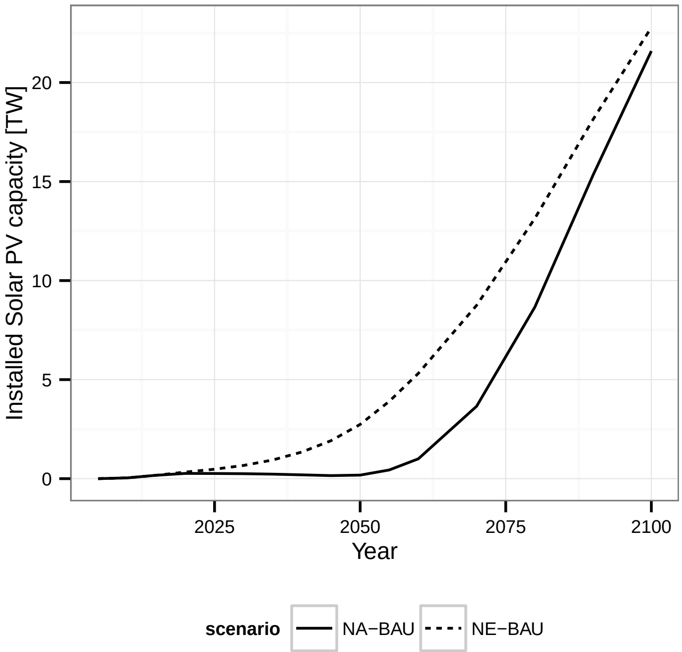


**Fig. 8** Solar Photovoltaic (PV) capacity paths in the Nash (NA-BAU) and the Negishi (NE-BAU) solution of the business-as-usual scenario.

## 6 Conclusions

We present an implementation of a decentralized solution algorithm to find a general equilibrium solution of an IA model of climate change mitigation. The main contribution is a more effective computation of trade interactions. We demonstrate robustness of the solution by comparing it to a solution from an established Negishi solution algorithm. The new algorithm – called Nash algorithm – has two major advantages: first, it is computationally more effective, and allows for an increased number of regions in the problem. The median run time of the model is reduced by more than a factor of 10 with respect to the Negishi algorithm. The new algorithm can also take advantage of parallel computing. Second, the algorithm allows, based on the assumption of decentralized actors, for an extended representation of real-world inter-regional externalities. In an exemplary application of the two algorithms to climate change mitigation, we demonstrate moderate differences in technology choice between a cooperative and non-cooperative solution: internalizing global spillover externalities related to investments into learning technologies accelerates the adoption of emerging low-carbon technologies such as solar power.

Further research could apply the Nash algorithm in model settings including other externalities, as, for example, the climate externality or technological spillover driven by R&D investments. A larger gap between the Nash and the Negishi solution can be expected in both cases. Moreover, the question of how these externalities could be explicitly internalized by policy instruments within the decentralized Nash solution algorithm arises. While corresponding instruments (e.g. carbon tax, technology subsidy) are well-understood conceptually, the design and implementation of solution algorithms of large-scale IA models will likely be challenged by each additional inter-regional externality and policy instrument.

## References


1. Arrow, K.J., Hahn, F.H. (1971). General competitive analysis. Mathematical Economic Texts, No. 6. Oliver and Boyd. San Francisco, pp. 452.
2. Bauer, N., Edenhofer, O., Kypreos, S. (2008). Linking Energy System and Macroeconomic Growth Models. Journal of Computational Management Science 5, 95-117.
3. Bosetti, V., Carraro, C., Massetti, E., Tavoni, M. (2006). A World Induced Technical Change Hybrid Model. Energy Journal, Special Issue #2, 13-37.
4. Bosetti, V., Carraro, C., Galeotti, M., Massetti, E., Tavoni, M. (2008). International energy R&D spillovers and the economics of greenhouse gas atmospheric stabilization. Energy Economics 30, 2912-2929.
5. Brechet, T., Camacho, C., Veliov, V.M. (2014). Model predictive control, the economy, and the issue of global warming. Annals of Operations Research 220, 25-48.
6. Brooke, A., Kendrick, D., Meeraus, A. (1992). GAMS - A User's Guide, Release 2.25. The Scientific Press, San Francisco.
7. Clarke, L., Jiang, K. et al. (2014). Assessing transformation pathways. In: Climate Change 2014 - Mitigation of Climate Change, Working Group III Contribution to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, chapter 6.
8. Copeland, B.R., Taylor, M.S. (2005). Free trade and global warming: a trade theory view of the Kyoto protocol. Journal of Environmental Economics and Management 49, 205-234.
9. Crassous, R., Hourcade, J.-C., Sassi, O. (2006). Endogenous Structural Change and Climate Targets, Modelling experiments with Imaclim-R. Energy Journal, Special Issue #1, 259-276.
10. Dixon, P.B. (1975). The Theory of Joint Maximization. Amsterdam: North-Holland Publishing Company.
11. Drud, A. (1994). CONOPT - A large-scale GRG code. ORSA Journal on Computing 6, 207-216.
12. Facchinei, F., Kanzow, C. (2010). Generalized Nash Equilibrium Problems. Annals of Operations Research 175, 177-211.
13. Finus, M., Rundshagen, B., Eyckmans, J. (2014). Simulating a sequential coalition formation process for the climate change problem: first come, but second served? Annals of Operations Research 220, 5-23.
14. Fudenberg, D., Levine, K.L. (1988). Open-Loop and Closed-Loop Equilibria in Dynamic Games with Many Players. Journal of Economic Theory 44, 1-18.
15. Huebler, M., L. Baumstark, M. Leimbach, N.Bauer, O. Edenhofer (2012). An integrated assessment model with endogenous growth. Ecological Economics 83, 118-131.
16. Kicsiny, R., Varga, Z., Scarelli, A. (2014). Backward induction for a class of closed-loop Stackelberg games. European Journal of Operational Research 237, 1021-1036.
17. Leimbach, M., Baumstark, L. (2010). The impact of capital trade and technological spillovers on climate policies. Ecological Economics 69, 2341-2355.
18. Leimbach, M., Bauer, N., Baumstark, L., Edenhofer, O. (2010). Mitigation costs in a globalized world: climate policy analysis with REMIND-R. Environmental Modeling and Assessment 15, 155-173.
19. Leimbach, M., Baumstark, L., Luderer, G. (2015). The role of time prefences in explaining the long-term pattern of international trade. Global Economy Journal 15, 83-106, DOI: 10.1515/gej-2014-0035.
20. Luderer, G., Leimbach, M., Bauer, N., Kriegler, E., Baumstark, L., Bertram, C., Giannousakis, A., Hilaire, J., Klein, D., Levesque, A., Mouratiadou, I., Pehl, M., Pietzcker,

R., Piontek, F., Roming, N., Schultes, A. and Schwanitz, V. J., Strefler, J. (2015). Description of the REMIND Model (Version 1.6) (November 30, 2015). Available at SSRN: http://ssrn.com/abstract=2697070.
21. Magne, B., Kypreos, S., Turton, H. (2010). Technology Options for Low Stabilization Pathways with MERGE. Energy Journal 31, 83-107.
22. Mandel, A., Gintis, H.(2016). Decentralized Pricing and the equivalence between Nash and Walrasian equilibrium. Journal of Mathematical Economics 63, 84-92.
23. Manne, A.S., Mendelsohn, M., Richels, R. (1995). MERGE - A Model for Evaluating Regional and Global Effects of GHG Reduction Policies. Energy Policy 23, 17-34.
24. Manne, A.S., Richels, R. (2004). The impact of learning-by-doing on the timing and costs of $CO_2$ abatement. Energy Economics 26, 603-619.
25. Manne, A. S., Rutherford, T. (1994). International trade, capital flows and sectoral analysis: formulation and solution of intertemporal equilibrium models. In: Cooper, W. W. and Whinston, A. B. (eds.), New directions in computational economics, Kluwer Academic Publishers, Dordrecht, The Netherlands, 191-205.
26. Mas-Colell, A., Whinston, M.D., Green J.R. (1995). Microeconomic Theory. Oxford University Press, New York.
27. Meinshausen, M., Raper, S. C. B., Wigley, T. M. L. (2011). Emulating Coupled Atmosphere-Ocean and Carbon Cycle Models with a Simpler Model, MAGICC6 Part 1: Model Description and Calibration. Atmos. Chem. Phys. 11: 141756, doi:10.5194/acp-11-1417-2011.
28. Messner, S., Schrattenholzer, L. (2000). MESSAGE-MACRO: Linking an energy supply sodel with a macroeconomic module and solving it iteratively. Energy 25, 267-282.
29. Negishi, T. (1972). General Equilibrium Theory and International Trade. North-Holland, Amsterdam.
30. Nordhaus, W.D, Boyer, J. (2000). Warming the World: Economic Models of Global Warming. MIT Press, Cambridge, Massachusetts.
31. Nordhaus, W.D., Yang, Z. (1996). A Regional Dynamic General-Equilibrium Model of Alternative Climate-Change Strategies. American Economic Review 86, 741-765.
32. Pan, V. (1992). Complexity of Computation with Matrices and Polynomials. SIAM Review 34, 225-262.
33. Rao, S., Peppo, I., Riahi, K. (2006). Importance of Technological Change and Spillovers in Long-term Climate Policy. Energy Journal, Special Issue #1, 123-139.
34. Rogelj, J., Hare, W., Lowe, J., van Vuuren, D.P. et al. (2011). Emission pathways consistent with a 2°Cglobal temperature limit. Nature Climate Change 1, 413-418.
35. Scarf, H. (1967). On the computation of equilibrium prices. In: W. Fellner (ed.), Ten Economic Studies in the Tradition of Irving Fisher. New York: John Wiley & Sons. pp.207-230.
36. Yang, Z. (2003). Reevaluation and renegotation of climate change coalitions - a sequential closed-loop game. Journal of Economic Dynamics & Control 27, 1563-1594.
37. Weber, C.L., Peters, G.P. (2009). Climate change policy and international trade: Policy considerations in the US. Energy Policy 37, 432-440.
38. Weyant, J., Davidson, O., Dowlabathi, H., Edmonds, J., Grubb, M., Parson, E.A., Richels, R., Rotmans, J., Shukla, P.R., Tol, R.S.J., Cline, W., Fankhauser, S. (1996). Integrated Assessment of Climate Change: An Overview and Comparison of Approaches and Results. In: J. P. Bruce et al. (eds.), Climate Change 1995: Economic and Social Dimensions of Climate Change. Cambridge University Press, pp. 367-439.