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Centennial to millennial geomagnetic secular variation

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SUMMARY

A time varying spherical harmonic model of the paleomagnetic field for 0-7 ka is used to investigate large scale global geomagnetic secular variation on centennial to millennial scales. We study dipole moment evolution over the past 7 kyr, and estimate its rate of change using the Gauss coefficients of degree 1 (dipole coefficients) from the CALS7K.2 field model and by two alternative methods that confirm the robustness of the predicted variations. All methods show substantial dipole moment variation on time scales ranging from centennial to millennial. The dipole moment from CALS7K.2 has the best resolution and is able to resolve the general decrease in dipole moment seen in historical observations since about 1830. The currently observed rate of dipole decay is underestimated by CALS7K.2, but is still not extraordinarily strong in comparison to the rates of change shown by the model over the whole 7kyr interval. Truly continuous phases of dipole decrease or increase are decadal to centennial in length rather than longer term features. The general large scale secular variation shows substantial changes in power in higher spherical harmonic degrees on similar time scales to the dipole. Comparisons are made between statistical variations calculated directly from CALS7K.2 and longer term paleosecular variation models: CALS7K.2 has lower overall variance in the dipole and quadrupole terms, but exhibits an imbalance between dispersion in g_2^1 and h_2^1 suggestive of long term non-zonal structure in the secular variations.

Key words: Geomagnetic dipole moment; Paleo-secular variation; Paleointensity; Archaeomagnetism; Geomagnetic field models;

1 INTRODUCTION

Global time-varying field models based on current and historical geomagnetic observations have been widely used to study the geomagnetic secular variation on decadal to centennial time scales (e.g., Bloxham et al. 1989; Bloxham & Jackson 1992; Jackson et al. 2000; Hulot et al. 2002; Finlay & Jackson 2003). Such secular variation studies have naturally focussed on the physical processes that are considered dominant on these relatively short time scales, leading to an emphasis on linking secular variation to decadal changes in length of day, geomagnetic jerks, torsional oscillations, searches for evidence for Ohmic diffusion, and the nature of fluid flow at the core-mantle boundary. The picture that emerges for the past 400 years is of stable large-scale spatial structure in the geomagnetic field, and of secular variation that appears more intense and smaller in both spatial and temporal scale in longitudes extending from about 100W to 120E than in the Pacific region. The anomalous behaviour in the Pacific has been correlated with seismic evidence for thermal anomalies in the lower-most mantle (e.g., Masters et al. 1996), and has been interpreted as a sign that geographically heterogeneous thermal and possibly chemical boundary conditions may have a detectable influence on the long term (millennial to million year) spatial structure and/or secular variation and reversals of the geomagnetic field (Cox & Doell 1964; Bloxham & Gubbins 1987; Laj et al. 1991; Johnson & Constable 1998; Gubbins 1998; Costin & Buffett 2004; Gubbins & Gibbons 2004). The detection of anomalous time-averaged fields and secular variation from the paleomagnetic record during the period 0-5 Ma remains a topic for current debate (e.g., McElhinny 2004).

Since systematic global measurements were begun about 175 years ago there has also been a steady decrease in the dipole strength at a rate of about 5% per century. The decrease may be linked mechanistically to geomagnetic reversals (Gubbins 1987), and has lead to some concerns that we might be observing the early stages of a reversal (Olson 2002; Hulot et al. 2002). Although the rate is rapid (about 5 times as fast) compared with that expected for free decay of the field if the geodynamo ceased operation, the historical record is too short to allow inferences about continued decay of the field or the likelihood of geomagnetic reversal based on these data alone. Two other sources of information that can be exploited for this purpose are the output from numerical dynamo simulations and paleomagnetic observations. Both of these are in principle also useful for studies of longer term geomagnetic secular variation on a broad range of spatial and temporal scales.

The current dipole decay rate and accompanying increase in complexity of the geomagnetic field do agree with features shown by some numerical dynamo models in the early stages of reversal. However, even though some of these numerical dynamos show Earth-like field behaviour, their parameter ranges remain far from those considered appropriate for Earth (Dormy et al. 2000) and the agreement may be coincidental. The physical origins of geodynamo variations on many time-scales are still not well understood (Hollerbach 2003), and it remains a significant challenge to relate the output of numerical simulation to paleomagnetic observations (see Kono & Roberts (2002) for a review). Any comparisons must rely on the evaluation of statistical properties of the paleofield and the simulations. The most fruitful of those currently being explored consider the symmetry properties of the geodynamo (originating from work by Young (1974) and Gubbins (1975) and reviewed in the paleomagnetic context by Gubbins (1998)), including symmetry or antisymmetry of the paleofield about the equator (McFadden et al. 1988; Tauxe & Kent 2004), axial symmetry (Constable & Johnson 1999), and the more general symmetry properties considered in some detail by Hulot & Bouligand (2005) (see also Bouligand et al. (2005)). In these comparisons the global paleofield variations are generally described in terms of the statistical distributions of the spherical harmonic coefficients that represent the spatial structure of the field, the distributional variability arising from the temporal variations of the paleomagnetic field (Constable & Parker 1988b).

In this work we use a recently developed time-varying paleomagnetic field model, CALS7K.2 (Korte & Constable 2005b), to study both dipole moment variability and long wavelength secular variation on centennial to millennial time scales, with a view to bridging the gap between the historical record and statistical studies on million year time scales.

In section 2 we discuss the results of a high-resolution study of the past variability of the geomagnetic dipole moment. The temporal development in terms of splines allows a direct estimate of the rate of change of dipole moment, with higher resolution than earlier paleomagnetic results derived solely from virtual axial dipole moments (VADMs) (McElhinny & Senanayake 1982; Yang et al. 2000). Detailed studies of the dipole moment allow the evaluation of the current dipole moment decrease in a longer term context, and are also essential for studies of solar variability and paleoclimate based on past cosmogenic nuclide production (e.g. Elsasser et al. 1956; Stuiver et al. 1991; Solanki et al. 2004).

Section 3 is concerned with analysis of the large scale secular variation manifested by CALS7K.2, including terms up to degree 5. We consider the distribution of power in the secular variation in both time and space and the evidence for non-zonal structure in the geomagnetic field and its secular variation. The statistical and symmetry properties of the Gauss coefficients from CALS7K.2 are compared with those from two recent paleosecular variation models.

2 DIPOLE MOMENT VARIABILITY

Details of the CALS7K.2 model have been presented elsewhere (Korte & Constable 2005b), so here we simply note that the modelling techniques are similar to those commonly used for the modern field (Bloxham & Jackson 1992; Jackson et al. 2000). CALS7K.2 is a spatially and temporally regularised spherical harmonic model extending to degree and order 10, and its temporal development is parametrised by B-splines with a knot spacing of 55 years. The model covers the time interval 5000 BC to 1950 AD, and is based on directional data derived from lake sediments and both directional and absolute paleointensity data from archaeological artefacts and young lava flows (Korte et al. 2005). Spatial and temporal regularisation is necessary to avoid overfitting of data with high uncertainties and spurious structure in areas sparsely covered by data particularly the southern hemisphere. As a consequence, the temporal and spatial resolutions of CALS7K.2 are no better than 100 years and degree and order 5, respectively, with short term and small scale field features being smoothed significantly by the regularisation. The notation used here is such that at any time t, CALS7K.2 is described by a suite of time varying Gauss coefficients $g_l^m(t)$ and $h_l^m(t)$ with degree l, $1 \leq l \leq 10$, and order m, $0 \leq m \leq l$.

At present a centred dipole with a moment of $7.78 \cdot 10^{22}\mathrm{Am}^2$ and tilted by about 11° relative to the Earth's axis accounts for 93.7% of the power observed at the Earth's surface. In the geomagnetic context we take power to mean the average of the squared magnetic field strength, B^2, over Earth's surface. This can be specified as a function of spherical harmonic degree and is then commonly refered to as the spatial geomagnetic power spectrum (Lowes 1974). CALS7K.2 allows us to make direct comparisons with the results obtained by current models, and we find that averaged over the past 7 kyr the dipole remains dominant contributing almost 98% of average power. In the past ancient dipole strength has commonly been studied using VADMs, obtained from paleomagnetic intensity results (Valet 2003). VADMs do not take account of non-dipole field contributions and to minimise that influence they are usually averaged over at least a few centuries. The temporal resolution of VADM studies of this kind for the past 12 millennia (McElhinny & Senanayake 1982; Yang et al. 2000) is therefore limited, and as we recently showed (Korte & Constable 2005a), the resulting VADM estimates are also systematically higher than the dipole moment from CALS7K.2. In that work we were able to show that there is no inconsistency between the two sets of results: the differences are attributable to data quality and to non-dipole field contributions that are aggravated by the available geographical sampling and persist in the averaged VADMs. In this section we investigate rates of change of the dipole moment using both the dipole moment from CALS7K.2 and the VADMs that have traditionally been used in such analyses. Note that strictly speaking we should compare VADMs to the purely axial dipole moment or use VDMs, virtual dipole moments taking into account the dipole tilt, for comparison to the full dipole moment. However, the differences both between dipole moment and axial dipole moment and between VDMs and VADMs are small in general and insignificantly small with respect to the resolution of the CALS7K.2 model.

2.1 Virtual axial dipole moments

We begin by seeking a high resolution estimate of temporal changes in the VADM, because the commonly used averaging of individual VADMs over several centuries significantly limits the temporal resolution of the result. First, we fit a smoothing spline (Constable & Parker 1988a) to the VADMs determined from the subset of 3092 data points from the archeointensity compilation used in CALS7K.2, according to the following transformation

$$VADM = \frac{4\pi a^3}{\mu_0} B(1 + 3\cos^2\theta)^{-\frac{1}{2}} \qquad (1)$$

with $a = 6371.2$ km the average radius of the Earth, $\mu_0 = 4\pi \cdot 10^{-7}$Vs/(Am) the permeability of free space, and θ the colatitude of the location for which the field strength B was obtained. The maximum temporal variability that the smoothing spline can represent is governed by the number of knot points and the quality of the fit to the data. The spline should represent all variation required by the data, and the variability should not be limited by the knot point interval. The chosen interval of ~ 23 years ensures this. The averaging of the non-dipole field in this case comes from the quasi-global coverage, and the temporal resolution achieved by the regularisation of the spline.

Two spline fits to the VADM data are presented here. A weighted spline was obtained with the data weighted by their uncertainty estimates, which were determined from the total uncertainty estimates (data and dating) of the intensity data used in producing the CALS7K.2 model. The spline was fitted with a normalised rms misfit of 1.0, i.e. the data were fitted to the expected value of χ^2 based on their uncertainties. In order to evaluate the influence of our uncertainty estimates we also fitted an unweighted spline for which the root mean square (rms) misfit was chosen to roughly reflect the standard deviation of the 1000 year averages and is $1.66 \cdot 10^{22}\,\mathrm{Am}^2$. Both these splines are shown together with the individual VADMs and their 1000 and 500 year averages for the intervals 5000 BC – 2000 BC and 1000 BC – 1950 AD, respectively, in Fig. 1a. Weighting the VADMs by their uncertainty estimates decreases the magnitude of the resulting curve in the time span of highest dipole moment, where the scatter and error estimates are highest. The main temporal structure, however, is robust and (apart from the expected bias in magnitude caused by influence of non-dipole contributions and uncertainties in the data) agrees reasonably well with the dipole moment M predicted from CALS7K.2, namely

$$M(t) = \left[\frac{4\pi a^3}{\mu_0} \sqrt{(g_1^0(t))^2 + (g_1^1(t))^2 + (h_1^1(t))^2} \right]. \qquad (2)$$

Both splines and the dipole moment are displayed in Fig. 1b together with VADM results predicted from the CALS7K.2 model at times and locations of the data points. VADM estimates (black dots, red and blue line) are almost all significantly higher than the dipole moment prediction from the spherical harmonic model CALS7K.2 (black line), although the VADM results (gray) predicted from the model at times and locations of data points mainly agree with VADM spline estimates. The main differences occur when the dipole moment is low, but non-dipole contributions are high (Korte & Constable 2005b).

2.2 Spherical harmonic descriptions

One of the significant improvements in the CALS-family models compared to previous modelling efforts for these time spans is the use of a physical regularisation instead of a simple truncation of the spherical harmonic series at low degrees and orders. However, it is possible to model the data allowing only a dipole in the spatial representation, i.e. a truncation of the spherical harmonic expansion right after degree and order one. Although this violates the philosophy of finding out how much spatial field structure is required by the data, a dipole only inversion can be a useful exercise in studying dipole variability and the influence of the non-dipole contributions. It avoids the use of spatial regularisation and prevents this from influencing the allowed temporal variability in the dipole. We therefore decided to try this exercise. We use the depleted data set underlying CALS7K.2 (after the iterative data rejection, see Korte & Constable (2005b)) for a dipole only inversion. The modelling method is the same as used for CALS7K.2, except that the spatial expansion is truncated at degree and order one and consequently no spatial damping is necessary. The knot-point spacing of the temporal splines and the weighting of the intensity data are the same as for CALS7K.2. An rms misfit of 1.0 could not be achieved with such a simple model (indicating the need for non-dipole terms in the field description) and the normalised rms misfit of the estimate shown in Fig. 2(a) is 1.3, resulting from 4 iterations with a temporal damping factor of 0.1 $\mathrm{nT}^{-2}\mathrm{yr}^4$.

To determine the reliability of each of the spherical harmonic dipole moment estimates and temporal variation we applied a bootstrap resampling method to the modelling. Based on the original depleted data set underlying CALS7K.2 3000 datasets with an identical number of data randomly drawn from the original set were created. The bootstrap considers each datum separately in the resampling approach (so that it is likely that for lake sediments at most a handful of data might be omitted from each location, but not a whole time series), and this allows the ratio of directional and intensity data to vary slightly among the bootstrap sample data sets.

All data sets were modelled with the same parameters and numbers of iterations as for the original CALS7K.2 and dipole only models respectively. The standard deviations obtained this way for the dipole moment are quite small, as shown in Fig. 2a. They represent uncertainty estimates based on the information content contained in the currently available data and probably are somewhat optimistic.

There is good agreement among the commonly used average VADMs, our spline fit and the dipole moment obtained from this intensity and directional data fit to a time-varying, tilted dipole. The truncation of this spherical harmonic expansion at degree and order one biases the result by spatial aliasing, which is a comparable effect to the influence of the non-dipole contributions in VADMs. The centennial-scale variability of our various dipole moment predictions is quite robust, although the temporal resolution is not as good as in modern geomagnetic field models.

The use of cubic splines for the temporal parametrisation allows a direct calculation of the rates of change in the various dipole estimates, and these are plotted together in Fig. 2b. We expect that among these estimates CALS7K.2 gives the most reliable estimate for dM/dt: nevertheless there is a surprising level of agreement among all the estimates. The lower resolution achievable in paleomagnetic versus historical models is clearly seen from the rates of change in Fig. 2b in comparison to those from the 1590-1990AD GUFM model (Jackson et al. 2000) in the overlapping time interval. Among the continuous paleomagnetic estimates, only CALS7K.2 is able to resolve at all the current decrease of the dipole seen since 1840 in the historical model GUFM (Jackson et al. 2000) as well as in other models from current data. The disagreement between the CALS7K.2 and GUFM dipole moments around 1850 is almost certainly due to this limited resolution and the fact that CALS7K.2 simply does not fully resolve a temorary dipole maximum. The CALS7K.2 dipole prediction in Fig. 2a shows that between about 1650 and 1800 there is an interruption in the steady decrease of dipole moment, with the rate of change still being close to 0 around 1840. This result has been confirmed in a recent study by (Gubbins et al. 2006), who estimated a constant rate of change in the axial dipole coefficient that is indistinguishable from zero for the time span 1590 to 1990. This result is derived directly from paleointensity data and differs considerably from the extrapolation used in GUFM for that time interval. Although the estimation techniques differ significantly, the paleointensity data and associated error estimates used in that study are exactly the same as in the equivalent time interval of CALS7K.2.

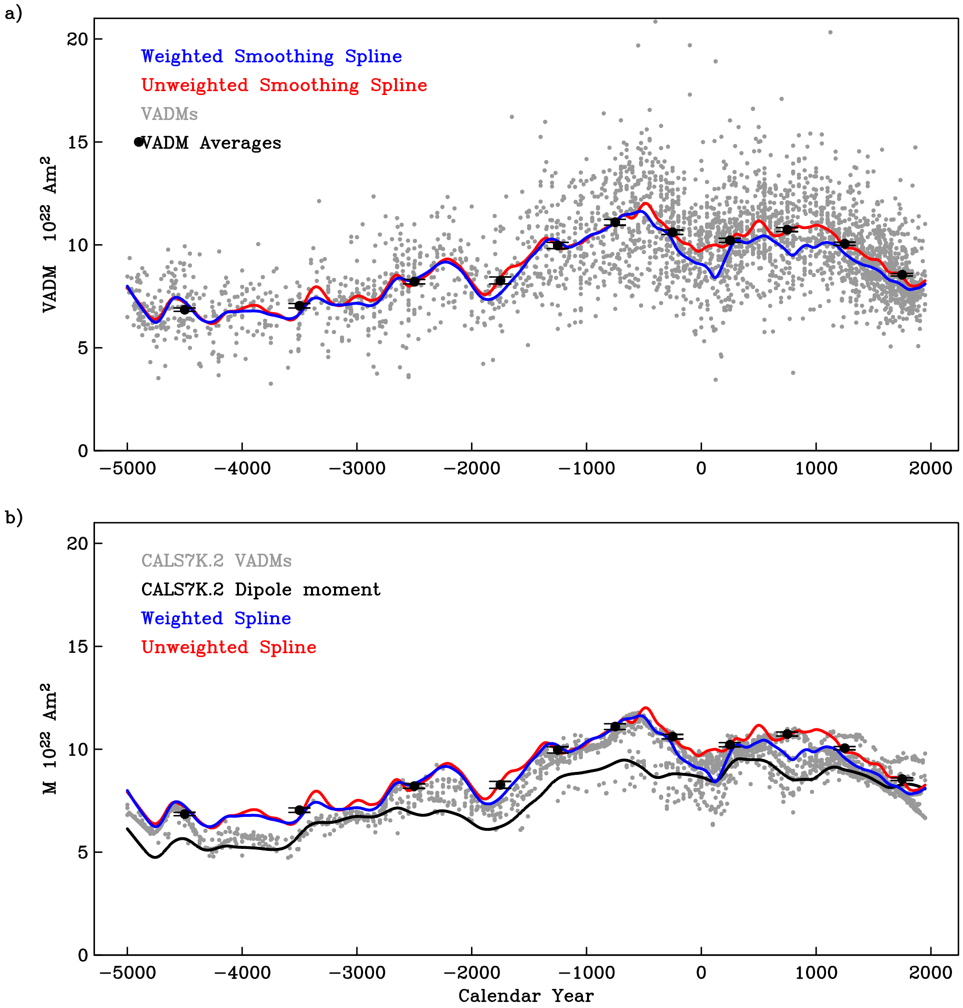

Figure 1. Virtual axial dipole moments (VADMs). **a** Individual VADM results (gray) used in CALS7K.2 and various dipole moment estimates based on them: 1000 and 500 year averages (black, 5000BC-2000BC and 1000BC-1950AD respectively) with one standard error uncertainties in the mean, spline fit (red) to the VADMs, spline fit to the VADMs (blue) weighted by the uncertainty estimates described in Korte et al. (2005). **b** VADM estimates (black dots, red and blue line) are almost all significantly higher than the dipole moment prediction from the spherical harmonic model CALS7K.2 (black line), although the VADM results (gray) predicted from the model at times and locations of data points mainly agree with VADM spline estimates. The main differences occur when the dipole moment is low, but non-dipole contributions are high (Korte & Constable 2005b).

3 LARGE SCALE SECULAR VARIATION FOR 0-7 KA

3.1 Spatial power variability

We turn now to an investigation of the large scale secular variation of the field, including the non-dipole contributions to CALS7K.2. We noted earlier that secular variation causes temporal changes in the spatial power distribution (Korte & Constable 2005b), so we begin by looking at how R_l, the spatial power at each spherical harmonic degree, varies with time t. R_l is the average squared field B at spherical harmonic degree l at Earth's surface (Lowes 1974), that is

$$R_l(t) = < \vec{B}_l(t)\cdot\vec{B}_l(t) >_{r=a} = (l+1)\sum_{m=0}^{l}[(g_l^m(t))^2+(h_l^m(t))^2]. \quad (3)$$

We limit this analysis to no more than degree 5, beyond which the model clearly lacks resolution and the regularisation dominates.

Fig. 3a shows the temporal development of power in each spherical harmonic degree at Earth's surface, together with the mean spectrum and the dispersion about the mean. The dispersion which reflects the amplitude of the variations is just the standard deviation of each R_l about its mean, but should not be confused with an uncertainty estimate. It is more or less proportional to l, but relative to the mean value is smallest in the dipole and strongest in the octupole (Table 1). The increasing similarity in temporal evolution of R_l at higher degrees (a feature which continues in degrees 6 to 10 which are not shown here) must be attributed to the regularisation in the model. The variability in degree 4 and 5 may already be damped to a certain degree by the spatial regularisation, which means that while the relative dispersion in the quadrupole clearly is lower than in the octupole we cannot say this with certainty about the higher degrees. Keep in mind also that we are looking only at the past 7k years here. Over (much) longer periods the dispersion in the dipole must be higher to allow for field reversals.

In Fig 3b the secular variation power in each degree with time is displayed and the temporal variability is characterised by the secular variation spectrum, again including the variance about the mean power in each degree. Note that while Fig. 2b and Fig 3b both contain information about the amount of change in the field

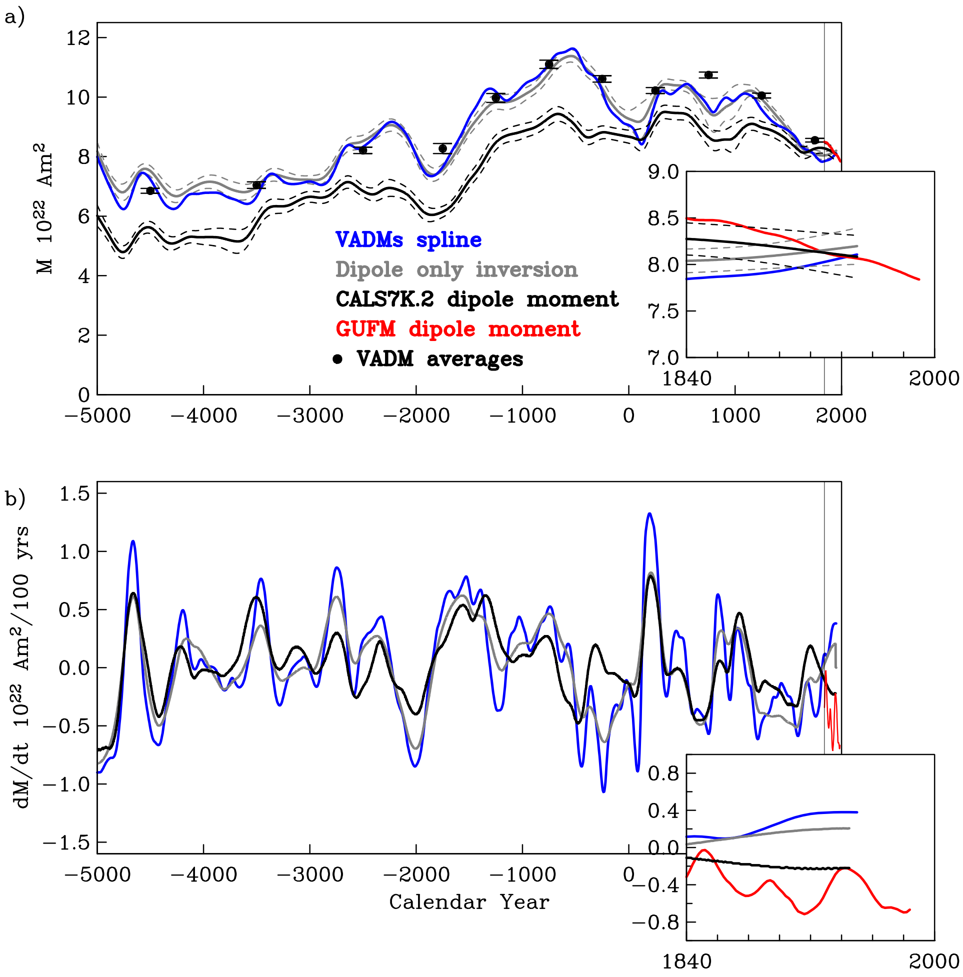

Figure 2. Dipole moment variability. **a** Dipole moment estimates (M) and **b** their rates of change for the past 7000 years obtained from the same data by different methods: average VADMs (black dots) and weighted spline fit (blue line) from Figure 1, spherical harmonic inversion of intensity and directional data for a dipole only (gray line), dipole moment estimate from CALS7K.2 (black line) and in the expanded right part also from the historical model GUFM (red, (Jackson et al. 2000)). Uncertainty estimates for the spherical harmonic models were determined by a bootstrap method, with the dashed lines giving one standard error. The agreement in most of the temporal structure is confirmed by the general accordance between rates of change per century for VADMs spline and spherical harmonic dipole moments in **b**.

per time interval, they were obtained in different ways. The rate of change of the dipole moment M shown in the previous section is given by

$$\frac{dM}{dt} = \frac{d}{dt}\left[\frac{4\pi a^3}{\mu_0}\sqrt{(g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2}\right], \qquad (4)$$

while the secular variation power S_l in each degree l is

$$S_l =< \frac{\vec{B}_l(t)}{dt}\cdot\frac{\vec{B}_l(t)}{dt} >_{r=a} = (l+1)\sum_{m=0}^{l}\left[\left(\frac{d(g_l^m)}{dt}\right)^2 + \left(\frac{d(h_l^m)}{dt}\right)^2\right]. (5)$$

The secular variation power thus includes no information about the sign of the field change. It is obvious that the dipole secular variation $S_1(t)$ is at least as dynamic as that of the higher degrees on all time scales of variation, and Table 1 shows that relative to the mean it is highest.

Table 1. Mean spectral powers R_l and S_l with dispersions as a measure for variability of the field and secular variation respectively.

Deg. l	R_l (nT^2)	Disp. R_l (%)	S_l ((nT/yr)^2)	Disp. S_l (%)
1	1.70828*10^9	38	410	95
2	2.105*10^7	58	263	90
3	1.254*10^7	84	123	73
4	2.65*10^6	68	29	53
5	4.9*10^5	53	5	63

3.2 Statistical models

Looking at the power and variability per spherical harmonic degree as we have done so far seems reasonable because the degree determines the spatial wavelength. Partitioning the power as a function of l according to what is seen in the modern field is also a strategy that has been widely used in modelling the paleosecular variation (PSV) on very long time scales. PSV studies are usually based on global collections of paleomagnetic directions derived from lava flows whose precise temporal ordering is often unknown. It is not possible to derive a time dependent spherical harmonic model like CALS7K.2 from these data. Instead the model is described by a collection of Gaussian distributions, one for each individual spherical harmonic coefficient: the mean of each distri-

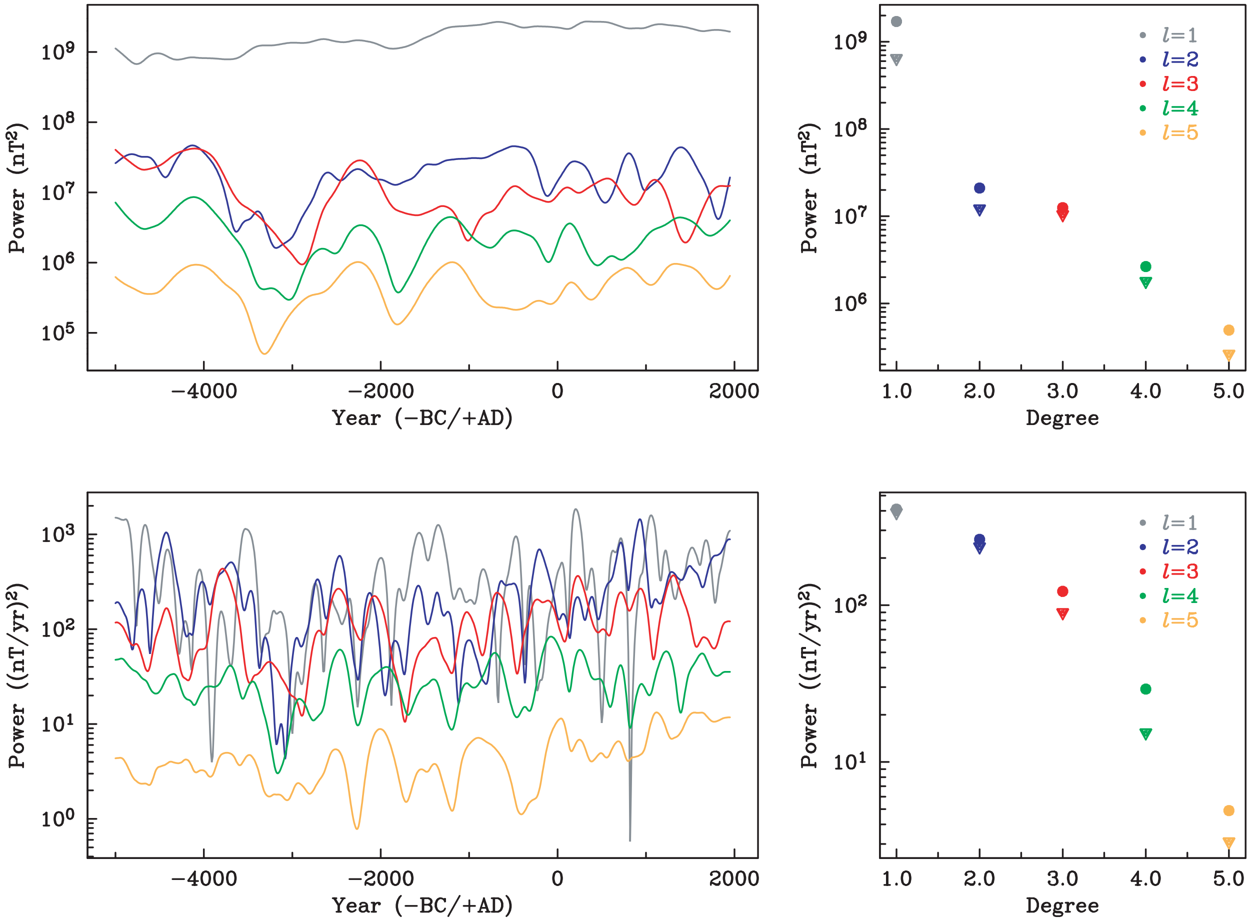

Figure 3. (a) Temporal development of degree l power in CALS7K.2 together with the power spectrum at Earth's surface including the dispersion (triangles) about the mean (dots). (b) The same for secular variation power.

bution gives the long term average for the spherical harmonic coefficient, and the variance about the mean represents the overall magnitude of the time variations without regard for any specific ordering. Such models, termed Giant Gaussian Processes (GGPs) (Constable & Parker 1988b), have generally disregarded any temporal correlations and frequency content in the signal, although in principle it is possible to include temporal correlations in such a process (Hulot & LeMouël 1994). Various simplifying assumptions can be made in constructing GGPs involving the symmetry of the geomagnetic field, and then tested for compatibility with the observations. Spherical symmetry is easily rejected in both the average and PSV because of the dominance of the axial dipole. Two other symmetries that have received consideration are equatorial and axisymmetry. See Hulot & Bouligand (2005) for an up to date discussion.

Equatorially symmetric (ES) contributions to the field potential are described by spherical harmonics with $l - m$ even, and equatorially antisymmetric (EA) by $l - m$ odd. One recent PSV model, TK03 (Tauxe & Kent 2004), supposes that the ES and EA terms have different amounts of variability, but each follows a specific functional form corresponding to a spatial power spectrum that is white over all degrees at the core-mantle boundary. The dispersions for TK03 up to degree and order 3 are shown in Fig. 4, where it is seen that the antisymmetric terms ($l - m$ odd) have a dispersion 3.8 times as large as the symmetric ones for the same degree l. In axisymmetric models (like TK03) all pairs of non-zonal Gauss coefficients with the same degree l and order m have the same dispersion. The CJ98 model (Constable & Johnson 1999) also shown in Fig. 4 is axisymmetric too, but in common with earlier models (e.g. Quidelleur & Courtillot 1996; Hulot & Gallet 1996; Kono & Tanaka 1995) follows a different approach from TK03. Deviations from a white spectrum are introduced for specific Gauss coefficients in order to fit the observations: In CJ98 this results in high dispersion in g_1^0, g_2^1 and h_2^1, and relatively low values in g_1^1 and h_1^1. Although the dispersions for these two models appear quite different, they fit the paleomagnetic directional data from lavas about equally well in part because the magnitude of the standard deviation or dispersions is effectively scaled by the average size of g_1^0 (see Table 2).

With CALS7K.2 we can estimate the statistics for the Gauss coefficients directly, so in Fig. 4 we compare the dispersion of the individual dipole, quadrupole and octupole coefficients of CALS7K.2 with those for the two statistical paleomagnetic models TK03 and CJ98. The dispersion, in fact again the standard deviation of the coefficients' time series, is a measure of the amplitude of variations in each coefficient. The dispersion in dipole and quadrupole terms is generally lower in CALS7K.2 than in the paleomagnetic models, which might reflect the shorter time interval of only 7k years in CALS7K.2 versus millions of years in the statistical models. The dispersion in CALS7K.2 might also be damped by the regularisation due to the lower temporal resolution, poor data distribution and lower data quality, but we would expect this to

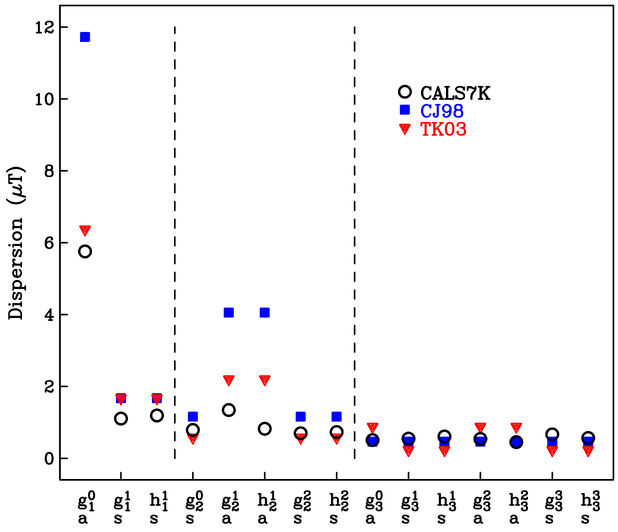

Figure 4. Dispersion in spherical harmonic coefficients of CALS7K.2 (Korte & Constable 2005b) and the statistical models CJ98 (Constable & Johnson 1999) and TK03 (Tauxe & Kent 2004) for dipole, quadrupole and octupole. The annotations a and s denote equatorially antisymmetric and symmetric coefficients.

have a stronger influence on the higher degrees and consider it to be of only minor influence in dipole and quadrupole. CALS7K.2 in dipole and quadrupole clearly shows similar dispersions to the statistical models with the highest dispersion in the axial dipole and high dispersion in the $m = 1$ coefficients for both degrees. It is not clear, however, whether the behaviour of CALS7K.2 is closer to CJ98 or TK03. In the octupole the dispersion in the CALS7K.2 coefficients is not higher in the EA than the ES parts, but neither is it uniform across all values of m. A striking difference is seen between the dispersions of the quadrupole order 1 coefficients, which are equal for g_2^1 and h_2^1 in the statistical models but not in CALS7K.2, which might reflect axial asymmetry in the field evolution. Such axial asymmetry was included in CJ98.nz, a non-zonal version of CJ98, but was not required to fit the paleomagnetic data used in those models. The average values of the coefficients themselves, listed in Table 2, are a measure for the deviation of the time-averaged field from an axial dipole. Most of the average coefficients except for the axial dipole are small in CALS7K.2: we notice the largest value occurs for g_2^0, which implies a zonal structure and (as noted by Hulot & Bouligand (2005)) equatorial symmetry breaking when combined with the average axial dipole field. The observed value for g_2^0 is close to that used in the CJ98 model.

3.3 Spectral content of CALS7K.2

To gain better insight into the time scales of large scale secular variation we calculated power spectral densities as a function of frequency for each $R_l(t)$ time series and for the individual Gauss coefficients up to the axial octupole g_3^0. We cannot make any physical interpretations at periods less than 55 years (0.018 yr^{-1}), the knot-point spacing of the temporal spline basis functions, so the spectra displayed in Fig. 5 are cut off at 0.018 yr^{-1}. The frequency spectra for R_l (Fig 5a) show generally similar, quite rapid fall-off for each degree. The flattening of the higher degree spectra at high frequency is due to the beginning influence of the knot spacing. No special periodicities are detected but octupole and higher degrees have most power at periods between 1000 and 2000 years. As we saw earlier, the temporal power is roughly proportional to the spatial power, except for quadrupole and octupole. In the quadrupole, there is less power in the periods of a few millennia and significantly more power at about 500 years. Looking at the temporal spectra of individual Gauss coefficients (Fig 5b), however, reveals a more complicated field behaviour: the axial dipole (g_1^0) also has most power at periods of 1000 to 2000 years, which is not seen in the complete, tilted dipole because the equatorial dipole contributions have most power at long periods with a continuous fall-off towards higher frequencies. The zonal contributions and h_2^1 are responsible for the high power around a period of 1000 years in quadrupole and octupole.

An approach which could lead to a better understanding of secular variation mechanisms is to look for frequency-dependent correlations among the coefficients used to describe the field. This could help to identify axial or zonal symmetries or systematic changes like westward drift which might allow inferences about the underlying physical processes. For this purpose we used the variation of the minimum bias multi-taper spectral methods described by Riedel & Siderenko (1995) and used in the paleomagnetic context (see Constable et al. (1998) and Constable & Johnson (2005)) to study power spectra and coherence among various rock- and paleomagnetic properties of marine sediment cores. The results we obtain from the analysis of CALS7K.2 are as follows: First we checked for large scale symmetries, using spatial power time series for EA and ES, zonal and non-zonal, and dipole and non-dipole contributions. No significant coherence exists between zonal and non-zonal field contributions but we noted a weak coherence above the significance level between ES and EA parts of R_l at periods of about 80 and 130 years. Both the zonal and EA field contributions are strongly dominated by the axial dipole and it turns out that the observed coherence is purely between axial dipole and ES field contributions, there is no coherence between non-axial dipole EA and the ES contributions. Coming back to the zonal field contributions, the correlation analysis shows coherence well above the significance level between the axial dipole power and the non-dipole zonal power, but only for most of the period range between 400 and 100 years and not the longer periods. Looking once more at individual coefficients instead of degree power, it turns out this is a correlation only between dipole and quadrupole zonal contributions, which are in phase over the period range from 400 down to the resolution limit of 55 years. Coherences in the same period range are also found between axial dipole (g_1^0) and g_2^1, g_3^1, respectively. Coherences above the significance level on the century time-scale also exist between g_1^1 and g_2^1, h_1^1 and h_2^1, g_2^1 and g_3^1, h_2^1 and h_3^1, g_2^1 and g_3^2, h_2^1 and h_3^2. Such correlations among variations in the Gauss coefficients have been contemplated in discussions of PSV models (Hulot & Gallet 1996).

4 CONCLUSIONS

4.1 Dipole secular variation

Our comparison between dipole and higher degree variation has shown that significant short-period variations exist in the geomagnetic dipole moment and even in the axial dipole, which had not been resolved by the previously used VADMs averaged over 500 to 1000 years (McElhinny & Senanayake 1982; Yang et al. 2000). An implicit assumption in using VADMs averaged over a few hundred years to represent actual dipole behaviour is that the variability of the non-dipole contributions is greater on short time scales than

Table 2. Average coefficients and their dispersion in μT for CALS7K.2 and two statistical models.

Coeff.	CALS7K.1 Average	Dispersion	CJ98 Average	Dispersion	TK03 Average	Dispersion
g1 0	-28.6	5.76	-30	11.72	-18	6.37
g1 1	-0.33	1.11	0	1.68	0	1.68
h1 1	0.51	1.19	0	1.68	0	1.68
g2 0	-1.66	0.79	-1.5	1.16	0	0.58
g2 1	-0.24	1.34	0	4.06	0	2.20
h2 1	-0.01	0.82	0	4.06	0	2.20
g2 2	0.42	0.70	0	1.16	0	0.58
h2 2	0.23	0.73	0	1.16	0	0.58
g3 0	-0.14	0.51	0	0.46	0	0.88
g3 1	-0.24	0.55	0	0.46	0	0.23
h3 1	-0.50	0.61	0	0.46	0	0.23
g3 2	0.33	0.54	0	0.46	0	0.88
h3 2	-0.19	0.45	0	0.46	0	0.88
g3 3	-0.39	0.67	0	0.46	0	0.23
h3 3	-0.55	0.57	0	0.46	0	0.23

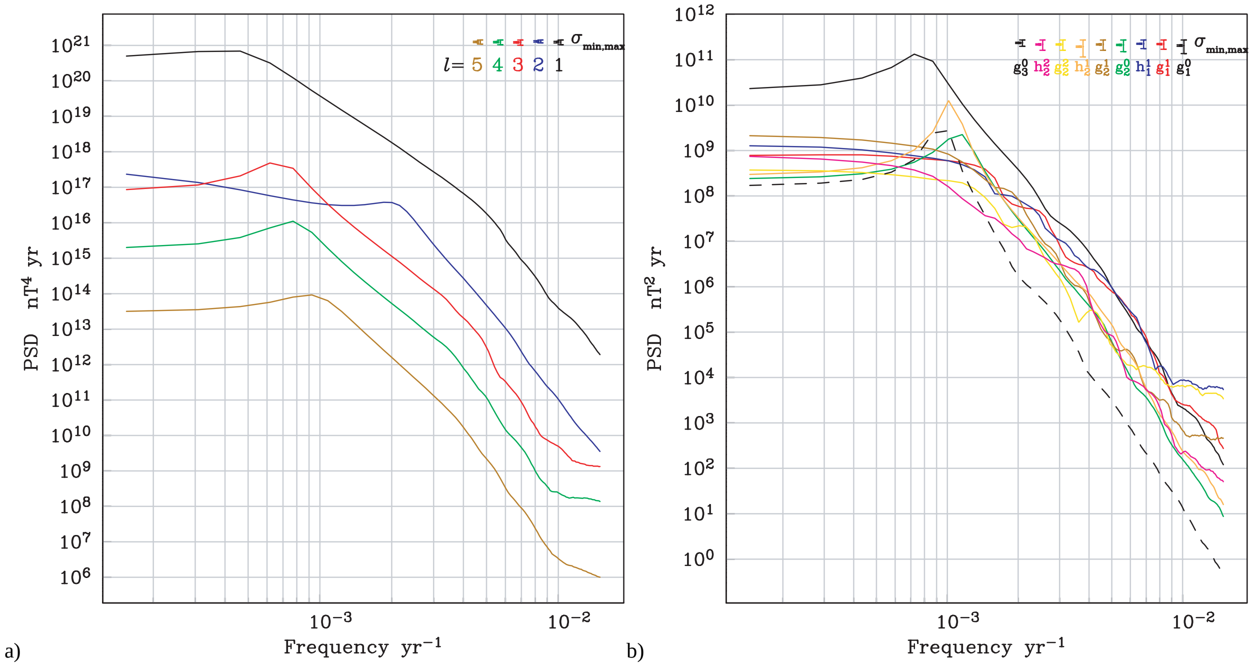

Figure 5. Power spectral densities of spherical harmonic degree *l* power (a) and individual coefficients (b) of CALS7K.2. The spectra are cut off at the period of 55 years, the knot-point spacing of the temporal spline basis functions.

that of the dipole itself, and that temporal averaging will efficiently remove the non-dipole contributions. Our analysis has shown that this is not the case. This shorter term variability of the dipole moment is important, and should for example be taken into account in studying solar irradiation and climate changes from cosmogenic nuclide production rates.

The dipole moment during the past 7000 years shows a distinctive long-term behaviour, with low values in the earlier half and high values in the recent half of that period, giving an average close to the current value. The values in the earlier half, however, are close to the long-term paleomagnetic average VADMs of 5.9×10^{22}Am^2 for the past 800 kyr (Valet 2003) and 4.5×10^{22}Am^2 for the past 160 Myr (Tauxe 2006). One cannot discount the possibility that the long-term average VADMs also suffer from unknown bias. The current dipole moment decrease is far from monotonic, but appears to be part of a general process lasting for about 2650 or 1650 years now, depending on how we judge the period around 0 AD between the two maxima. The rates of change (Fig. 2b) vary significantly and truly continuous decreases or increases are decadal to centennial, rather than millennial features. The average rate of decrease since 350 AD is about 1.5% per century, significantly lower than the current 5% per century and only slightly higher than the free decay rate of about 1% per century. The rate for the current decrease is strong, but easily within the range observed during the past seven millennia. A continuing nearly linear decay over more than a few centuries seems unlikely, as there is only one interval in which the average field increases over more than 1000 years. Future improvements in resolution may lead to revisions of the longevity of this feature.

4.2 Secular variation features of quadrupole and octupole field contributions

The lower dispersion in dipole and quadrupole coefficients in CALS7K.2 than in statistical paleomagnetic models might reflect the shorter time interval of CALS7K.2. This implies that lower frequency variations exist and contradicts a conclusion by Hongre et al. (1998) that the typical time scales involved in spherical harmonic degree 2 (and 3) are of the order of a few centuries only and there is no correlation over more than about 450 years. Our inference is supported by the result that axial quadrupole, octupole and h_2^1 show significant power at periods of about 1000 years. The time interval of 2000 years studied by Hongre et al. (1998) might just have been too short to reveal periodicities longer than half a millennium. However, our analysis does confirm that g_2^0 does not average to zero over several millennia as had been found by Hongre et al. (1998) and is compatible with what is seen in time-averaged paleomagnetic models. This means that the averaged field is not simply equatorially antisymmetric but zonal structure and equatorial symmetry breaking have to be expected in long-time averages.

4.3 Symmetries on millenial and paleomagnetic time scales

A clear difference between the 7000 year model CALS7K.2 and the longer term PSV models is seen in the dispersions of the order 1 quadrupole coefficients, which are unequal for g_2^1 and h_2^1 in CALS7K.2. While the PSV data have been explained by a zonal average model, the millennial scale model requires some axial asymmetry, which might reflect the low secular variation observed in the Pacific region and imply a time-scale of at least a few millennia for this feature. The correlation seen between g_1^0 and g_2^1, g_3^1 respectively also could be an indication for this persistent asymmetry, of which we cannot tell the time-scale. Further coherences between individual coefficients of CALS7K.2 are rather inconclusive and hard to interpret in terms of symmetry. As they are rather weak they may not prove robust over longer time-scales.

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